

# Glossary

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**To be completed.**

## *Ausdehnungslehre*

The term *Ausdehnungslehre* is variously translated as 'extension theory', 'theory of extension', or 'calculus of extension'. Refers to Grassmann's original work and other early work in the same notational and conceptual tradition.

## **Bivector**

A *bivector* is a sum of simple bivectors.

## **Bound vector**

A *bound vector*  $\mathbf{B}$  is the exterior product of a point and a vector.  $\mathbf{B} = \mathbf{P} \wedge \mathbf{x}$ . It may also always be expressed as the exterior product of two points.

## **Bound bivector**

A *bound bivector*  $\mathbf{B}$  is the exterior product of a point  $\mathbf{P}$  and a bivector  $\mathbf{W}$ :  $\mathbf{B} = \mathbf{P} \wedge \mathbf{W}$ .

## **Bound simple bivector**

A *bound simple bivector*  $\mathbf{B}$  is the exterior product of a point  $\mathbf{P}$  and a simple bivector  $\mathbf{x} \wedge \mathbf{y}$ :  $\mathbf{B} = \mathbf{P} \wedge \mathbf{x} \wedge \mathbf{y}$ . It may also always be expressed as the exterior product of two points and a vector, or the exterior product of three points.

## **Cofactor**

The *cofactor* of a minor  $M$  of a matrix  $A$  is the signed determinant formed from the rows and columns of  $A$  which are not in  $M$ .

The sign may be determined from  $(-1)^{\sum (r_i + c_i)}$ , where  $r_i$  and  $c_i$  are the row and column numbers.

## **Dimension of a linear space**

The *dimension of a linear space* is the maximum number of independent elements in it.

## **Dimension of an exterior linear space**

The *dimension of an exterior linear space*  $\Delta_m$  is  $\binom{n}{m}$  where  $n$  is the dimension of the underlying linear space  $\Delta_1$ .

The dimension  $\binom{n}{m}$  is equal to the number of combinations of  $n$  elements taken  $m$  at a time.

**Dimension of a Grassmann algebra**

The *dimension of a Grassmann algebra* is the sum of the dimensions of its component exterior linear spaces.

The dimension of a Grassmann algebra is then given by  $2^n$ , where  $n$  is the dimension of the underlying linear space.

**Direction**

A *direction* is the space of a vector and is therefore the set of all vectors parallel to a given vector.

**Displacement**

A *displacement* is a physical interpretation of a vector. It may also be viewed as the difference of two points.

**Exterior linear space**

An *exterior linear space* of grade  $m$ , denoted  $\Delta_m$ , is the linear space generated by  $m$ -elements.

**Force**

A *force* is a physical entity represented by a bound vector. This differs from common usage in which a force is represented by a vector. For reasons discussed in the text, common use does not provide a satisfactory model.

**Force vector**

A *force vector* is the vector of the bound vector representing the force.

**General geometrically interpreted 2-element**

A *general geometrically interpreted 2-element*  $\mathbf{U}$  is the sum of a bound vector  $\mathbf{P} \wedge \mathbf{x}$  and a bivector  $\mathbf{W}$ . That is,  $\mathbf{U} = \mathbf{P} \wedge \mathbf{x} + \mathbf{W}$ .

**Geometric entities**

Points, lines, planes, ... are *geometric entities*. Each is defined as the space of a geometrically interpreted element.

A point is a geometric 1-entity.

A line is a geometric 2-entity.

A plane is a geometric 3-entity.

**Geometric interpretations**

Points, weighted points, vectors, bound vectors, bivectors, ... are *geometric interpretations* of  $m$ -elements.

**Geometrically interpreted algebra**

A *geometrically interpreted algebra* is a Grassmann algebra with a geometrically interpreted underlying linear space.

**Grade**

The *grade* of an  $m$ -element is  $m$ .

The grade of a simple  $m$ -element is the number of 1-element factors in it.

The grade of the exterior product of an  $m$ -element and a  $k$ -element is  $m+k$ .

The grade of the regressive product of an  $m$ -element and a  $k$ -element is  $m+k-n$ .

The grade of the complement of an  $m$ -element is  $n-m$ .

The grade of the interior product of an  $m$ -element and a  $k$ -element is  $m-k$ .

The grade of a scalar is zero.

(The dimension  $n$  is the dimension of the underlying linear space.)

**GrassmannAlgebra**

The concatenated italicized term *GrassmannAlgebra* refers to the *Mathematica* software package which accompanies this book.

**A Grassmann algebra**

A *Grassmann algebra* is the direct sum of an underlying linear space  $\Lambda_1$ , its field  $\Lambda_0$ , and the exterior linear spaces  $\Lambda_m$  ( $2 \leq m \leq n$ ).

$$\Lambda_0 \oplus \Lambda_1 \oplus \Lambda_2 \oplus \dots \oplus \Lambda_m \oplus \dots \oplus \Lambda_n$$

**The Grassmann algebra**

The *Grassmann algebra* is used to describe that body of algebraic theory and results based on the *Ausdehnungslehre*, but extended to include more recent results and viewpoints.

**Intersection**

An *intersection* of two simple elements is any of the congruent elements defined by the intersection of their spaces.

**Laplace expansion theorem**

The *Laplace expansion theorem* states: If any  $r$  rows are fixed in a determinant, then the value of the determinant may be obtained as the sum of the products of the minors of  $r$ th order (corresponding to the fixed rows) by their cofactors.

**Line**

A *line* is the space of a bound vector. Thus a line consists of all the (perhaps weighted) points on it and all the vectors parallel to it.

**Linear space**

A *linear space* is a mathematical structure defined by a standard set of axioms. It is often referred to simply as a 'space'.

**Minor**

A *minor* of a matrix  $\mathbf{A}$  is the determinant (or sometimes matrix) of degree (or order)  $k$  formed from  $\mathbf{A}$  by selecting the elements at the intersection of  $k$  distinct columns and  $k$  distinct rows.

***m*-direction**

An *m*-direction is the space of a simple *m*-vector. It is also therefore the set of all vectors parallel to a given simple *m*-vector.

***m*-element**

An *m*-element is a sum of simple *m*-elements.

***m*-plane**

An *m*-plane is the space of a bound simple *m*-vector. Thus a plane consists of all the (perhaps weighted) points on it and all the vectors parallel to it.

***m*-vector**

An *m*-vector is a sum of simple *m*-vectors.

***n*-algebra**

The term *n*-algebra is an alias for the phrase *Grassmann algebra with an underlying linear space of  $n$  dimensions*.

**Origin**

The *origin*  $\mathbf{O}$  is the geometric interpretation of a specific 1-element as a reference point.

**Plane**

A *plane* is the space of a bound simple bivector. Thus a plane consists of all the (perhaps weighted) points on it and all the vectors parallel to it.

**Point**

A *point*  $\mathbf{P}$  is the sum of the origin  $\mathbf{O}$  and a vector  $\mathbf{x}$ :  $\mathbf{P} = \mathbf{O} + \mathbf{x}$ .

**Point space**

A *point space* is a linear space whose basis elements are interpreted as an origin point  $\mathbf{O}$  and vectors.

**Point mass**

A *point mass* is a physical interpretation of a weighted point.

**Physical entities**

Point masses, displacements, velocities, forces, moments, angular velocities, ... are *physical entities*. Each is represented by a geometrically interpreted element.

A point mass, displacement or velocity is a physical 1-entity.

A force, moment or angular velocity is a physical 2-entity.

**Physical representations**

Points, weighted points, vectors, bound vectors, bivectors, ... are geometric *representations* of physical entities such as point masses, displacements, velocities, forces, moments.

Physical entities are *represented* by geometrically interpreted elements.

**Scalar**

A *scalar* is an element of the field  $\Delta_0$  of the underlying linear space  $\Delta_1$ .

A scalar is of grade zero.

**Screw**

A *screw* is a geometrically interpreted 2-element in a three-dimensional (physical) space (four-dimensional linear space) in which the bivector is orthogonal to the vector of the bound vector.

The bivector is necessarily simple since the vector subspace is three-dimensional.

**Simple element**

A *simple element* is an element which may be expressed as the exterior product of 1-elements.

**Simple bivector**

A *simple bivector*  $\mathbf{V}$  is the exterior product of two vectors.  $\mathbf{V} = \mathbf{x} \wedge \mathbf{y}$ .

**Simple  $m$ -element**

A *simple  $m$ -element* is the exterior product of  $m$  1-elements.

**Simple  $m$ -vector**

A *simple  $m$ -vector* is the exterior product of  $m$  vectors.

**Space of a simple  $m$ -element**

The *space of a simple  $m$ -element*  $\alpha_m$  is the set of all 1-elements  $\mathbf{x}$  such that  $\alpha_m \wedge \mathbf{x} = \mathbf{0}$ .

The space of a simple  $m$ -element is a linear space of dimension  $m$ .

**Space of a non-simple  $m$ -element**

The *space of a non-simple  $m$ -element* is the union of the spaces of its component simple  $m$ -elements.

**2-direction**

A *2-direction* is the space of a simple bivector. It is therefore the set of all vectors parallel to a given simple bivector.

**Underlying linear space**

The *underlying linear space* of a Grassmann algebra is the linear space  $\Lambda_1$  of 1-elements, which together with the exterior product operation, generates the algebra. The dimension of an underlying linear space is denoted by the symbol  $n$ .

**Underlying point space**

An *underlying point space* is an underlying linear space whose basis elements are interpreted as an origin point  $\mathbf{O}$  and vectors. It can be shown that from this basis a second basis can be constructed, all of whose basis elements are points.

**Union**

A *union* of two simple elements is any of the congruent elements which define the union of their spaces.

**Vector**

A *vector* is a geometric interpretation of a 1-element.

**Vector space**

A *vector space* is a linear space in which the elements are interpreted as vectors.

**Vector subspace of a geometrically interpreted underlying linear space**

The *vector subspace of a geometrically interpreted underlying linear space* is the subspace of elements which do not involve the origin.

**Weighted point**

A *weighted point* is a scalar multiple  $\mathbf{a}$  of a point  $\mathbf{P}$ :  $\mathbf{a P}$ .