3

# ROBUST DESIGN

## 3.1 INTRODUCTION

### 3.1.1 Background

At the end of the second world war Japan was left with the problem of recovering economically without quality tools or materials.

In order to develop quality products without quality tools and materials Dr. Genichi Taguchi, at the beginning of the electronics industry in Japan, developed the technique of *experimental robust design*. It is claimed by some sources that this techniques was responsible for some of the large gains in product quality experienced by that industry in the decades to follow.

In the early 1980's Xerox and AT&T Bell discovered Taguchi's work. Since then, most large manufacturers in the U.S. have been attempting to apply the concepts.

The methodology for the design of robust products is equally valid for the design of robust *processes*, and can be extended to include the design of *services*.

### 3.1.2 What is Robust Design?

A robust product is one that is *insensitive to variation*.

A robust design is one that is insensitive to material property and manufacturing tolerance variation to the extent that a very high proportion of the product is manufactured within specification and without the need for extensive inspection or adjustment.

A corollary of this is that a robust design permits the use of the least costly materials and manufacturing processes without compromising quality and performance.

A robust product is one that performs and continues to perform to the satisfaction of the customer despite the possible drifts (variation) in its design parameters expected over the normal product life.

### 3.1.3 Sources of variation

Common sources of variation are:

- Variation in material properties: density, yield strength, modulus of elasticity, homogeneity, contamination, …
- Variation due to manufacturing process variability: dimensions, heat treatment, residual stresses, …
- Variation due to degradation with time: wear, corrosion, embrittlement, …
- Variation due to environment: temperature, humidity, dust, supply voltage, electromagnetic fields, vibration, …

### 3.1.4 Why design robust products?

A robust product:

- Is of higher quality, and therefore continues to satisfy customer expectations of target performance under the intended operating conditions throughout the expected life of the product.
- Costs less to manufacture, since the cheapest materials and manufacturing tolerances may be used.
- Costs less to repair and service, since allowances for the effects of wear and degradation have been designed in to the product.
- Increases market share by minimising time to market through less “surprises” in the product development process.

Design for robustness is a central element in *design for quality*

- Examples: door fit on car, shoe sizes, television screen colour spectrum, voltage regulator, thermostat, safety valve,...

### 3.1.5 Comparison with other types of optimization

Traditional optimization is usually directed at maximising some performance parameter in a deterministic sense.

Pure robustification is directed at minimizing the *variation* of a performance parameter under the assumption that the designed nominal level of performance is satisfactory.

In general product optimization, the cost of variation (conformance to specification) is just one of the cost components among others (performance, material, manufacturing, …) to be minimised.

### 3.1.6 How are robust products designed?

Taguchi developed his robust design techniques purely from an *experimental* point of view. That is, he developed techniques by which experiments could be conducted efficiently on a range of prototypes to determine the ones with the least sensitivity to variation.

These experimental techniques can be time-consuming and expensive, since often many different prototypes need to be built. But they need to be used when no analytical prototypes (mathematical models) for the quality variables of the product exist (as is often the case in the electronics or process industries).

For mechanical systems it is more often the case that analytical prototypes can be constructed for the quality variables of interest. In this case it is quicker and much more cost effective to use an analytical approach. Additionally, more robust designs can be obtained as more of the solution space can be explored.

The analytical approach to robustification is several orders of magnitude "better" than an experimental approach in each of time, cost and accuracy. It should always be used in preference whenever an analytical model can be constructed.

This chapter explores the application of analytical robustification techniques to analytical prototypes.

**Problem 3.45 Sources of variation**

Discuss the types of design parameters that can cause variation in the performance of a product.

**Problem 3.46 Designing robust products**

Write brief notes on the advantages and disadvantages of designing robust products.

## 3.2 TERMINOLOGY

### 3.2.1 Failure modes

A *failure mode* of a product is any of the characteristic modes (ways) in which the product can fail to perform as intended.

In the development of any given type of product the technologist will usually identify, from experience, a set of failure modes of particular relevance to that type of product. These are the modes of failure most likely to occur and usually most difficult to control.

For example: an engineer responsible for developing the technology of a paper feeder to be used in a photocopier would know that the two major failure modes are *misfeeding* paper and *multifeeding* paper.

### 3.2.2 Design parameters

A *design parameter* is any of the physical quantities (variables) required to specify a design (dimensions, material properties, forces, …).

Design parameters may be further classified into three types: control parameters, noise parameters, and constant parameters.

- *Control parameters* are those design parameters over which the designer has specific control, that is, which have a range of feasible values, from which one must be chosen by the designer as the design value of the parameter.
- *Noise parameters* are those design parameters over which the designer has no direct control (applied load, input voltage, temperature, …). These parameters are often due to environmental variables, user input, or input parameters from a larger system.

- *Constant parameters* are those design parameters which the designer early in the process decides to fix at a constant value often either to simplify the design or to satisfy externally imposed constraints (environmental safety, material availability, manufacturing process availability, …).

### 3.2.3 Feasible domains

The *feasible domain* of a control parameter is the range of values in which its mean value is constrained to lie (generally by practical design or external system requirements).

### 3.2.4 Quality variables

A variable $z$ is a *quality variable* if it is a function of design parameters $x_1, x_2, x_3, \ldots$

$$z = g(x_1, x_2, x_3, \ldots)$$

and if one of the failure modes of the product has been quantified to depend on the value of $z$ lying outside a certain specified target range.

Put another way: A quality variable is a parameter which, if kept within its target range will ensure one aspect of the quality of the product.

This *target range* is generally determined by the technologist responsible for the design. The *target* is often set at the mid-point of the target range.

The essential characteristic of a quality variable is that the higher the probability that the value of the variable remains within the specified range over the life of the product, the higher the quality of the product.

Quality variables are often only under the control of the designer through knowledge of the functional relationships they have to the design parameters. That is, they are dependent rather than independent variables.

### 3.2.5 Critical parameters

In large systems the output (quality) variables of components can become the input (design parameters) of the system.

A *critical parameter* is a design parameter or a quality variable which has the properties that

- Its value is critical to the performance of the design
- Its value is often difficult to maintain at the value required to make the design work.

Critical parameters may also be viewed as those parameters that the product development process needs to track from design intent to the final manufactured product to ensure that they remain within specification throughout the various stages.

**Problem 3.47 Terminology for robust design**

Explain the terms *failure mode*, *control parameter*, *noise parameter*, *quality variable* and *critical parameter*.

## 3.3 THE CONCEPT OF ROBUSTNESS

*Exploit non-linearity!*

If the output of a non-linear system is to be as constant (least variation) as possible then we can use the non-linearity of the system operating characteristic to our advantage by placing the operating point where the gradient of the characteristic is smallest.

**NON-LINEAR SYSTEM CHARACTERISTIC**

### 3.3.1 The first order robustness problem

**MINIMISE VARIANCE WHILE KEEPING THE MEAN ON TARGET**

Suppose a quality variable $z$ expressed in terms of design parameters $x$ and $y$ by $z = g(x, y)$.

The design parameters $x$ and $y$ are considered to be independent random variables whose probability distributions have means $\mu_x$ and $\mu_y$ and variances $\nu_x$ and $\nu_y$.

The quality variable $z$ is also a random variable with mean $\mu_z$ and variance $\nu_z$ (dependent on the distributions of $x$ and $y$).

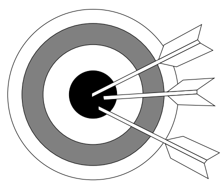
Suppose that to ensure that the product performs as intended, we need to keep the quality variable as close as possible to the target value $\tau_z$ despite the variation of $x$ and $y$.

The first order robustness problem is to determine, for given values of the variances $\nu_x$ and $\nu_y$, the values of $\mu_x$ and $\mu_y$ which minimize the variance $\nu_z$ while keeping $\mu_z$ on target $\tau_z$.

It is called the first order problem because it contains a number of "first order" approximations as part of the solution process. Extension to any number of parameters follows *mutatis mutandis*.

*For a given mean of the quality variable and variances of the design parameters, determine the means of the design parameters which minimize the variance of the quality variable.*

### 3.3.2 The general robustness problem



The general robustness problem is to determine, for given probability distributions of $x$ and $y$, the values of the means of $x$ and $y$ which minimize the unreliability associated with $z$. This general problem is quite complicated and will not be discussed further here.

*For given probability distributions of the design parameters, determine the means of the design parameters which minimize the unreliability associated with the quality variable.*

### 3.3.3 Visualization of the robustification process

In order to *visualise* the process of first order robustification in a non-trivial case we shall use Mathematica to work graphically through a simple two parameter problem.

The steps are as follows:

1. Write the model for the quality variable $z$ in terms of the design parameters $x$ and $y$.
2. Plot a three-dimensional graph of $z$ as a function of $x$ and $y$.
3. Slice the plot through at the target value $\tau_z$.
4. Read off the $(x,y)$ points on the cut (which give the target value).
5. Calculate an expression for the variance $v_z$.
6. Substitute given values for the variances $v_x$ and $v_y$ in the expression for $v_z$.
7. Substitute the $(x,y)$ points into the expression for $v_z$
8. Select the minimum value of the variance $v_z$
9. Identify which point(s) gave this minimum value.
10. Check that this solution keeps the mean of z on target.

**Problem 3.48 The concept of robustness**

Explain in your own words the concept of robustness using diagram(s) where appropriate.

**Problem 3.49 First order robustness problem**

What is the first order robustness problem?

**Problem 3.50 Exploitation of non-linearity**

A quality variable q is related to its one major design parameter x by $q = x^2(1-x)$.

If the target value for q is 0.05, and the feasible range of x is $0.1 \le x \le 1$, determine the value of x which will give the most robust design.

If x is uniformly distributed with support of 0.08 ($\mu_x \pm 0.04$), compare the variations in q due to the different possible settings of x.

Show these results on a diagram.

## 3.4 TAGUCHI'S QUALITY LOSS FUNCTION

Taguchi approaches the concept of robustness via the concept of the *quality loss function*.

### 3.4.1 One item

For one item the supposition is that the total quality loss (defined by Taguchi as the "loss to society") is proportional to the squared deviation from the target value for the quality variable

.

**QUALITY LOSS FOR ONE ITEM**

$$Q = k(y - \tau)^2$$

| | | | |
|---|---|---|---|
| where | $Q$ | is the quality loss | \$ |
| | $k$ | is a constant relating variability to loss | $\$/q^2$ |
| | $y$ | is the quality variable | q |
| | $\tau$ | is the target value for the quality variable | q |

To determine k we need one point on the graph.

If the quality loss is $Q_0$ at $y = \tau + \Delta$ then $k = \dfrac{Q_0}{\Delta^2}$

### 3.4.2 Many items

Suppose now that there are many items of the same sort produced in which the quality variable is distributed about its mean value $\mu$ .

The quality loss due to an item with value $y_i$ is

$$Q_i = k(y_i - \tau)^2$$

Averaging this for a large number of items yields

Derivation of the Quality Loss Function

$$Q = k[(\mu - \tau)^2 + \sigma^2]$$

where $\mu$ is the mean of the quality variable $y$ q

$\sigma^2$ is the variance of the quality variable $q^2$

**QUALITY LOSS FOR MANY ITEMS**

Hence, minimizing the average quality loss is equivalent to minimizing the sum of the variance and the square of the deviation of the mean from target.

In practice this is best done by:

- Setting and keeping the mean on target
- Minimizing the variance

### Problem 3.51 Derivation of average quality loss

Using Taguchi's assumption that the quality loss Q for an item is proportional to its squared deviation from the target value $\tau$, derive the expression for the average quality loss of a large number of similar product whose quality variable is random distributed with mean $\mu$ and standard deviation $\sigma$.

### Problem 3.52 Example of average quality loss

It has been determined that a new model of car jack is returned by customers when it fails to operate smoothly due to the width of the square thread being less than 3.8 mm or greater than 4.2 mm. Although a jack retails for only $50, the total cost to society of a returned jack is estimated at $250. Calculate the average quality loss per jack in a production run in which the thread width is distributed with mean 4.1 mm and standard deviation 0.05 mm.

Comment on your result.

## 3.5 FIRST ORDER ANALYTICAL ROBUST DESIGN

### 3.5.1 The algorithm

The stages in a first order analytical robust design are as follows:

1. Construct a design model for the quality variable of interest $z$ in terms of the design parameters $x_i$

$$z = g(x_1, x_2, \ldots, x_s)$$

2. Calculate the mean $\mu_z$ and variance $\nu_z$ of the quality variable by means of $\mu_z = g(\mu_{x1}, \mu_{x2}, \ldots, \mu_{xs})$ and

$$\nu_z = \left[\frac{\partial z}{\partial x_1}\right]_\mu^2 \nu_{x1} + \left[\frac{\partial z}{\partial x_2}\right]_\mu^2 \nu_{x2} + \ldots + \left[\frac{\partial z}{\partial x_s}\right]_\mu^2 \nu_{xs}$$

Remember that the derivatives are evaluated at the points $x_i = \mu_{xi}$ .

3. Put $\mu_z$ equal to the target value in the equation for the mean of Stage 2 and solve for one of the control parameters $\mu_{xa}$, say. This control parameter is called the adjustment parameter and is usually chosen as the easiest to solve for in terms of $\tau_z$ and the other $\mu_{xi}$.
4. Use the equation of Stage 3 to eliminate $\mu_{xa}$ from the equation for $\nu_z$ derived in Stage 2.
5. Find the values of the means of the control parameters $\mu_{xi}$ which will minimize the variance $\nu_z$ of the quality variable, but retaining the $\mu_{xi}$ within their feasible domains.
6. Calculate the corresponding value of the adjustment parameter.
7. Check the results.

The robustification process will need to search for a solution within the region of the control parameter space defined by the feasible domains of the control parameters.

### 3.5.2 Contribution analysis

With each design parameter is associated a term in the first order approximation to the variance equation called its *contribution* to the variance of the quality variable. The variance then becomes a simple sum of contributions, one from each parameter.

Since the variance equation can be complex, it is important to estimate the magnitude of these terms over the feasible domains of the design parameters relevant to the problem, and to rank them in order of contribution. If some of the terms can be shown to remain an order of magnitude less than others over these ranges, then they can most likely be neglected, at least in a first iteration.

### 3.5.3 Notes on methodology

- Real designs will usually involve a lot of parameters making minimisation difficult. Yet the more design parameters under the control of the designer, the more scope there is to make useful gains in robustness.

- Real designs have constraints on the values of their design parameters. Any equality constraints (rather than inequality) can be used to decrease the number of parameters involved in the minimisation.
- We may obtain the same result by minimising the quality loss function. However, the constraint (stage 3) that the mean shall remain precisely on target both reduces the dimension of the solution space by one, and makes the minimisation calculation more numerically stable.
- The variances of the design variables $x_i$ should be treated as constants in the minimization process. The objective of the robustification process is to minimize the quality loss function *without asking for higher (and hence more expensive) tolerances*.
- Some of the design parameters will be noise parameters and some will be constant parameters. Nevertheless, each will probably have a variance and should therefore be included in the calculation. (Pure numbers, like the number of tines in a fork will not have a variance).
- Sometimes simple inspection of the *form* of the variance will lead to immediately useful conclusions, and should always be tried first.
- Sometimes the form of the variance function will be such that the minimum variance occurs for very large or impractical values of the design parameters. This is useful information and indicates that either the feasible domain constraints need to be imposed or a cost equation should be formed which includes costs other than that due to variance.

**Problem 3.53 Robustification methodology**

Explain in your own words the procedure you would adopt and the formulae you would use for robustifying a design.

## 3.6 EXAMPLE: LOCATING STRUTS

Suppose that it is required to support a device at a precise height from a fixed base by a pair of struts. The positioning of the feet of the struts and the length of the struts have known fixed tolerances and corresponding variances.

What should be the best length of the struts (and hence positioning of the feet)?

Note that this is a purely geometric example and is not intended to consider stability or strength at this stage.

## 3.7 EXAMPLE: SIMPLE POWER MODEL

A quality variable $z$ is related to two design parameters $x$ and $y$ by the analytical model:

$$z = ax^m y^n$$

where $a$ is constant and $x$, $y$, $m$, and $n$ are positive.

By following the algorithm for first order robust design we can show that

- The relationship between the coefficients of variation of $x$ and $y$ which minimizes the variance of $z$ while keeping its mean on target is

$$m\hat{x}^2 = n\hat{y}^2 .$$

- The mean values of $x$ and $y$ which give the most robust design are

$$\mu_x = \left( \left(\frac{\tau_z}{a}\right) \left(\frac{m}{n}\frac{\nu_x}{\nu_y}\right)^{\frac{n}{2}} \right)^{\frac{1}{(m+n)}}$$

$$\mu_y = \left( \left(\frac{\tau_z}{a}\right) \left(\frac{n}{m}\frac{\nu_y}{\nu_x}\right)^{\frac{m}{2}} \right)^{\frac{1}{(m+n)}}$$

- The minimum variance of $z$ is

$$\nu_z\big|_{min} = m(n+m)\tau_z^2\hat{x}^2 = n(n+m)\tau_z^2\hat{y}^2$$

**Problem 3.54 Simple power model**

Derive the formulae of the previous section.

**Problem 3.55 Stiffness of a cantilever**

The stiffness of a cantilever spring has the model

$$k = \frac{YWH^3}{4L^3}$$

Suppose that $Y$ and $L$ are constant parameters and that $W$ and $H$ are control parameters.

Using the results obtained from the simple power model above:

1. Determine the relationship between the coefficients of variation of the control parameters which will ensure that $k$ has minimum variance.

2. Determine an expression for the best values of the control parameters.

3. Determine an expression for the minimum variance for $k$.

**Problem 3.56 Wooden cantilever**

A flexible element is required with a given stiffness characteristic. The proposed design consists of a single cantilever sawn from wood of rectangular cross-section where the force-deflection characteristics of its tip are used for the spring.

The following data is given:

The target stiffness is $\tau_k = 0.25$ N/mm

The control parameters are the width *W* and height *H* with tolerances ±3 mm.

The constant parameters are the Young's modulus of wood (Douglas Fir) 10 000 ± 3000 N/mm$^2$ and the length L of the beam 1000 ± 3 mm.

Using the results obtained from the previous example

1. Design a cantilever by choosing values of W and H to keep the mean of k on target. Making reasonable assumptions for estimating the variances from the tolerances calculate the standard deviation of the stiffness.

2. Determine the cross-section that will minimize the variance of the stiffness.

3. Calculate the resulting minimum standard deviation of the stiffness, and compare it to your first design.

4. Estimate the material savings.

**Problem 3.57 Helical spring**

A helical spring is to be designed for mass production with its deflection δ under force F to be as close as possible to a target specification of 5 mm.

The formula for the deflection of a helical spring is given by:

$$\delta = \frac{8FD^3N}{d^4G}$$

The data for the spring design as originally specified is given in the table below:

•

| PARAMETER | Sym | Nominal | Tolerance | Unit | Type |
|---|---|---|---|---|---|
| Coil Diameter | D | 20 | ± 3.0 | mm | Control |
| Number of Coils | N | 10 | ± 0.6 | 1 | Control |
| Applied Force | F | 50 | ± 9.0 | N | Noise |
| Wire Diameter | d | 3 | ± 0.06 | mm | Constant |
| Shear Modulus | G | 79 000 | ± 1200 | $N/mm^2$ | Constant |
| Deflection | $\delta$ | 5.00 | | mm | Quality |

**Table 7: Data for helical spring design**

Consider that the springs are to be used in a military device, and hence will be manufactured under military standards.

1. For the values in the above table, estimate a specification tolerance for the deflection outside of which you would expect 64 per million of the springs to lie.

Using the simple power model formulae

2. Determine the best nominal values for the control parameters for reducing the variation in the deflection. (Assume the tolerances remain fixed).

3. Determine the new specification tolerance for the deflection outside of which you would expect 32 per million of the springs to lie, and compare this to the original design.

4. Compare the cost of the material used between the original and the new design.

[Note carefully: These calculations do not take in to account other important design constraints, for example maximum stress. However, they do indicate the directions in which the nominal values of design parameters may be moved to minimize their effect on variability.]

# 3.8 CASE STUDY: PASSIVE FILTER NETWORK

## 3.8.1 Origin of the problem

The problem to be analysed is described in The Principles of Design by Nam P. Suh (Oxford University Press, 1990), and in a paper Using Taguchi Methods to Apply the Axioms of Design by Stephen F. Filippone in Robotics and Computer-Integrated Manufacturing (Vol. 6, No 2, 1989)

## 3.8.2 Statement of the problem

A passive filter network is to be designed to measure the displacement signal generated by a strain gauge transducer. The network provides the interface between the strain gauge transducer/demodulator and the recording instrument with a galvanometer/light-beam deflection indicator. The network conditions the signal generated by a strain gauge transducer with demodulated output and measures the original displacement signal by filtering out the carrier frequency.

Design the circuit to maintain the filter cut-off frequency and galvanometer full scale deflection as close as possible to their target values.

## 3.8.3 Approach

We outline the approach as follows:

1. Decide on the control parameters ($C, R_1, R_2$).

2. Write down the model for the filter cut-off frequency F.

3. Put F equal to its target value (6.84 Hertz) and solve for a control parameter (the easiest to solve for). In this case we choose C.

4.Write down the model for the galvanometer deflection X.

5. Put the X equal to its target value (3 inches) and solve for a control parameter (the easiest to solve for). In this case we choose $R_2$.

6. Derive the formulae for the first order approximations to the variances of F and X: $\nu_F$ and $\nu_X$.

7. Substitute the expressions for C and $R_2$ into the formulae for $\nu_F$ and $\nu_X$. The variances are now a function of one control parameter $R_1$.

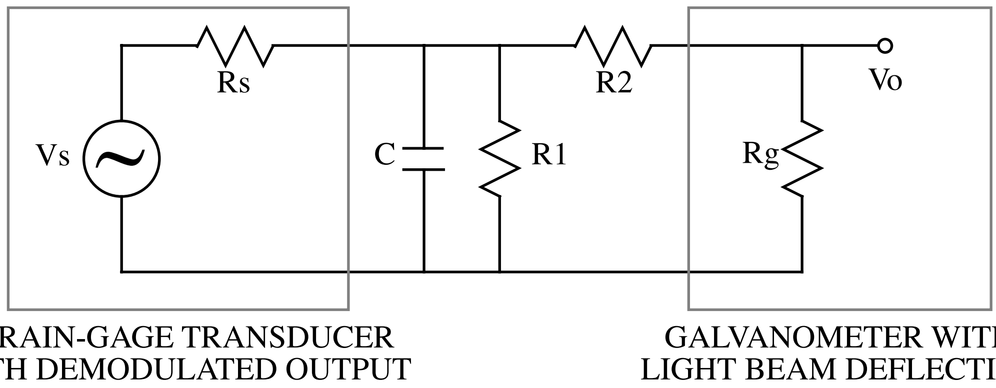
8. Define an overall quality loss $Q_T$ as a weighted sum of the vari-

ances of the quality variables: $\nu_F$ and $\nu_X$.

9. Find the value of $R_1$ which minimizes $Q_T$.

10. Substitute this value back to find the corresponding optimum values of C and $R_2$.

11. Check that these three optimum values keep the quality variables on target.




**RC FILTER NETWORK**

### 3.8.4 Nomenclature

| | | | |
|---|---|---|---|
| C | Filter capacitance | Control parameter | Farad |
| $R_1$ | Filter resistance | Control parameter | Ohm |
| $R_2$ | Filter resistance | Control parameter | Ohm |
| $R_g$ | Galvanometer resistance | Noise parameter | Ohm |
| $R_s$ | Strain gauge resistance | Noise parameter | Ohm |
| $V_s$ | Strain gauge voltage | Noise parameter | Volt |
| G | Galvanometer sensitivity | Noise parameter | Volt/in. |
| F | Filter cut-off frequency | Quality variable | Hertz |
| X | Galvanometer deflection | Quality variable | Inch |
| $F_t$ | Target frequency | Target | Hertz |
| $X_t$ | Target deflection | Target | Inch |
| $\nu_F$ | Variance of cut-off frequency F | Calculated | Hertz$^2$ |

| | | | |
|---|---|---|---|
| $v_X$ | Variance of deflection X | Calculated | $Inch^2$ |
| $k_1$ | Frequency variance weighting factor | | \$/ $Hertz^2$ |
| $k_2$ | Deflection variance weighting factor | | \$/ $Inch^2$ |
| $Q_T$ | Total quality loss | | \$ |

### 3.8.5 The filter cut-off frequency

The target cut-off frequency: *6.84 hertz*
The model

$$F = \frac{\frac{1}{R_s} + \frac{1}{R_1} + \frac{1}{R_g + R_2}}{2\pi C}$$

### 3.8.6 The galvanometer full-scale deflection

The target full-scale deflection: *±3 inches*
The model

$$X = \frac{R_g R_1 V}{G(R_s R_1 + (R_s + R_1)(R_g + R_2))}$$

### 3.8.7 The results

By plotting the graph of the total quality loss $Q_T$ against $R_1$ the minimum value was $0.135 at $R_1$ equal to 550 ohm.

The corresponding value of $R_2$ was 415 ohm, and of C was 282 microfarad.